

UK JUNIOR MATHEMATICAL CHALLENGE

TUESDAY 28TH MARCH 2000

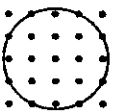
Organised by the **United Kingdom Mathematics Trust**
from the **School of Mathematics, University of Leeds**



SOLUTIONS LEAFLET

This solutions leaflet for the JMC is sent in the hope that it might provide all concerned with some alternative solutions to the ones they have obtained. It is not intended to be definitive. The organisers would be very pleased to receive alternatives created by candidates.

1. E Half of $999 = \frac{1}{2}(1000 - 1) = 500 - \frac{1}{2} = 499\frac{1}{2}$.
2. B $2000 - 1642 = 358$.
3. B $x = 50 - 22$. This method uses the theorem:
An exterior angle of a triangle is equal to the sum of the two interior and opposite angles.
4. D The values are A 24 B 14 C 14 D 36 E 21.
5. A If UKMT is reflected in a horizontal mirror line directly below it then image A appears.
6. D A van load can take only two crates and hence twelve van loads will be required.
7. C 12 hours and 15 minutes from 6:45 am will take the time to 7:00 pm and there will then be 29 minutes of the 12 hours and 44 minutes remaining.
8. C In C there is no connection between the pair of holes on the left of the card and the pair of holes on the right. There must, however, be such a connection for the display on the front of the card to be as shown.
9. E If three-quarters of the members are boys then one-quarter are girls and the number of boys is three times the number of girls.
10. B $140\,000 \times 6 \text{ g} = 840\,000 \text{ g} = 840 \text{ kg}$. *Almost one tonne!*
11. D The relevant years are 2, 11, 20, 101, 110, 200, 1001, 1010, 10101 and 1100.
12. D The total area of the four strips is 40 cm^2 , but there are four squares, each of area 1 cm^2 , where two strips overlap. Hence the area covered is $(40 - 4) \text{ cm}^2$.
13. B The girls who travel by bus make up 12% ($\frac{1}{4}$ of 48%) of the whole school and the corresponding figure for boys is 26% ($\frac{1}{2}$ of 52%). Hence 38% of the whole school travel by bus.
14. E In reverse, the stages which lead to 45 are: $45 \leftarrow 43 \leftarrow 34 \leftarrow 17$.
15. A Let Dilly's age be x . Then Dally is $x + 7$. In four years time Dilly will be $x + 4$ and Dally will be $x + 11$. Therefore $x + 11 = 2(x + 4)$ and hence $x = 3$. Dilly is 3, Dally is 10 and the sum of their ages is 13.
16. D The number of words = $256 \times 33 \times 9 \approx 250 \times 300 = 75\,000$. The best estimate, therefore, is 76 000.
17. D If a number is divisible by 9, then the sum of its digits must also be a multiple of 9. The sum of the digits of $d6d41$ is $2d + 11$, which must be an odd number between 13 and 29 inclusive. The only odd multiple of 9 in this interval is 27.
18. C Let the distance between adjacent dots be one unit.
Then a circle of radius $\sqrt{5}$ units whose centre is at the centre of the grid passes through eight dots, as shown.



19. B The difference between successive numbers in the list is $\frac{1}{3}(\frac{1}{3} - \frac{1}{5}) = \frac{1}{15}$.
Therefore $y = \frac{1}{3} - \frac{1}{15} = \frac{2}{5}$.

20. E The total cost of four apples, four oranges and four bananas is $\pounds 1.54 + \pounds 1.70 = \pounds 3.24$. Hence the amount Mr. Bean would pay for one apple, one orange and one banana is $\pounds 3.24 \div 4 = 81\text{p}$.
21. A Each hour, Tick's watch gains three minutes on Tock's watch. It will, therefore, take 20 hours before it is one hour ahead of it.
22. C Let the width and height of each block of wood be x and y respectively and the height of the table be h . Then: $h + x = y + 84$ and $h + y = x + 96$.
Add these two equations:
Subtracting $(x + y)$ from both sides:
$$h + x + h + y = y + x + 84 + 96$$
$$2h = 180$$
$$\therefore h = 90$$
23. A If 21 and 35 are factors of the number, then 3, 5 and 7 must all be included amongst its prime factors. This means that the required number must be a multiple of 105 and, as the complete list of factors of 105 is 1, 3, 5, 7, 15, 21, 35 and 105, the answer is 105.
(A number which is the product of three different prime numbers must have exactly eight factors. If p, q and r are all different prime numbers then the factors of pqr are 1, p, q, r, pq, pr, qr and pqr itself.)
24. B Exchanging the '2' and the '5', then the '2' and the '7' and, finally, the '2' and the '9' gives a display of 635792, which is a multiple of 4.
(A whole number is a multiple of 4 if, and only if, the number formed by its last two digits, in this case 92, is a multiple of 4. Can you prove this?)
25. E Comparing the leading diagonal and the second row:
 $13 + c + e = 5 + c + 15$ and hence $e = 7$.
Comparing the top row and the third column:
 $13 + a + b = b + 15 + e$ and hence $a = e + 2 = 9$.
Comparing the second row and the second column:
 $a + c + d = 5 + c + 15$ and hence $d = 20 - a = 11$.

13	a	b
5	c	15
x	d	e

We now have:

Let the 'magic' total be T .
Then: $T = 22 + b$
 $T = 20 + c$
 $T = 18 + x$
 $T = b + c + x$.

Adding the first three of these equations:

$$3T = 60 + b + c + x$$

$$\text{Hence: } 3T = 60 + T$$

$$\text{Thus: } T = 30.$$

$$\text{Therefore } x = 30 - 18 = 12.$$

13	9	b
5	c	15
x	11	7

(Notice that the magic total, in this case 30, is three times the number in the middle of the magic square, in this case 10. Can you prove that this is always the case in a 3×3 magic square?)