JMO mentoring scheme

October 2010 paper - answers

1. Ans : 17

2. Ans : 43

3. Because 3^m and 7^n are both odd numbers whatever m and n, so the difference must be even.

4. (i) By Pythagoras's rule, AB = $\sqrt{117}$ and BC = $\sqrt{52}$ so $AB^2 + BC^2 = 169 = AC^2$.
   (ii) By Pythagoras's rule, AB = $\sqrt{289}$ and BC = $\sqrt{100}$ so $AB^2 + BC^2 = 389 < AC^2$ (= 441).

5. Ans : Integers from 17 to 24.
   The factors pairs of 164 are (1, 164), (2, 82) and (4, 41). If there are an even number of integers, the average must be between the middle two integers, that is, the average is not an integer. This can be achieved with 8 numbers averaging 20½. There is no solution with an odd number of integers.

6. Ans : $(x, y) = (5, 15), (4, 6)$ or $(3, 3)$.
   Multiplying equation by $3xy$ gives $6y - 3x = xy$.
   Factor pairs of −18 are (−1, 18), (−2, 9), (−3, 6), (−6, 3), (−9, 2), (−18, 1)
   $x - 6 = -1, -2$ or -3 : we stop here otherwise $x - 6$ is not negative.

7. After the first move, all moves involve an even number of counters. Thus the originally empty pile must always contain an odd number of counters.

8. Ans : we would expect the majority vote to be correct 13 times out of 16 (reduced from 26 out of 32).
   Arrangements for A, B, C are : (0, 0, 0), (1, 0, 0), (0, 1, 0), (0, 0, 1), (0, 1, 1), (1, 0, 1), (1, 1, 0), (1, 1, 1)
   For (0, 0, 0) all will vote *against* more **1**s than **0**s which is correct.
   For (1, 0, 0), A will vote *against* more **1**s than **0**s, B's and C's vote will be random. The majority will only be incorrect if **both** B and C vote incorrectly. This will only happen 1 time out of 4.
   Similarly for (0, 1, 0) and (0, 0, 1).
   The same argument applies when there are two **1**s and one **0** but voting *for* more **1**s than **0**s.
   For (1, 1, 1) all will vote *for* more **1**s than **0**s which is correct.

9. Ans : $2\pi - 2\sqrt{3}$
   The sides of the equilateral triangle are 2. Its height is $\sqrt{3}$. Hence its area is $\sqrt{3}$. The area of one 60° sector is $\frac{1}{6} \times \pi \times 2^2 = \frac{2\pi}{3}$.
   Overlapping three of these includes the area we want but we have counts the equilateral three times instead of once.