**Compound and Double Angle Trig Formulae Past Paper Questions**

**Mark Scheme**

**June 2005**

| Question Number | Scheme | Marks |
|---|---|---|
| 5 (a) | $\cos 2A = \cos^2 A - \sin^2 A$ ( + use of $\cos^2 A + \sin^2 A \equiv 1$) | M1 |
| | $= (1 - \sin^2 A); - \sin^2 A = 1 - 2\sin^2 A$ (*) | A1 (2) |
| (b) | $2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv 4\sin\theta\cos\theta; -3(1 - 2\sin^2\theta) - 3\sin\theta + 3$ | B1; M1 |
| | $\equiv 4\sin\theta\cos\theta + 6\sin^2\theta - 3\sin\theta$ | M1 |
| | $\equiv \sin\theta(4\cos\theta + 6\sin\theta - 3)$ (*) | A1 (4) |
| (c) | $4\cos\theta + 6\sin\theta \equiv R\sin\theta\cos\alpha + R\cos\theta\sin\alpha$ | |
| | Complete method for $R$ (may be implied by correct answer) | |
| | [ $R^2 = 4^2 + 6^2$, $R\sin\alpha = 4$, $R\cos\alpha = 6$ ] | M1 |
| | $R = \sqrt{52}$ or 7.21 | A1 |
| | Complete method for $\alpha$; $\alpha = 0.588$ (allow 33.7°) | M1 A1 (4) |
| (d) | $\sin\theta\ (4\cos\theta + 6\sin\theta - 3) = 0$ | M1 |
| | $\theta = 0$ | B1 |
| | $\sin(\theta + 0.588) = \frac{3}{\sqrt{52}} = 0.4160..$ (24.6°) | M1 |
| | $\theta + 0.588 = (0.4291), 2.7125$ [or $\theta + 33.7° = (24.6°), 155.4°$] | dM1 |
| | $\theta = 2.12$ cao | A1 (5) |
| | | **[15]** |

**January 2006**

| Question Number | Scheme | Marks |
|---|---|---|
| **7.** | (a) (i) Use of $\cos 2x = \cos^2 x - \sin^2 x$ in an attempt to prove the identity. | M1 |
| | $\dfrac{\cos 2x}{\cos x + \sin x} = \dfrac{\cos^2 x - \sin^2 x}{\cos x + \sin x} = \dfrac{(\cos x - \sin x)(\cos x + \sin x)}{\cos x + \sin x} = \cos x - \sin x$ * cso | A1 **(2)** |
| | (ii) Use of $\cos 2x = 2\cos^2 x - 1$ in an attempt to prove the identity. | M1 |
| | Use of $\sin 2x = 2\sin x \cos x$ in an attempt to prove the identity. | M1 |
| | $\dfrac{1}{2}(\cos 2x - \sin 2x) = \dfrac{1}{2}(2\cos^2 x - 1 - 2\sin x \cos x) = \cos^2 x - \cos x \sin x - \dfrac{1}{2}$ * cso | A1 **(3)** |
| | (b) $\cos\theta(\cos\theta - \sin\theta) = \dfrac{1}{2}$ Using (a)(i) | M1 |
| | $\cos^2\theta - \cos\theta\sin\theta - \dfrac{1}{2} = 0$ | |
| | $\dfrac{1}{2}(\cos 2\theta - \sin 2\theta) = 0$ Using (a)(ii) | M1 |
| | $\cos 2\theta = \sin 2\theta$ * | A1 **(3)** |
| | (c) $\tan 2\theta = 1$ | M1 |
| | $2\theta = \dfrac{\pi}{4}, \left(\dfrac{5\pi}{4}, \dfrac{9\pi}{4}, \dfrac{13\pi}{4}\right)$ any one correct value of $2\theta$ | A1 |
| | $\theta = \dfrac{\pi}{8}, \dfrac{5\pi}{8}, \dfrac{9\pi}{8}, \dfrac{13\pi}{8}$ Obtaining at least 2 solutions in range | M1 |
| | The 4 correct solutions | A1 **(4)** |
| | If decimals $(0.393, 1.963, 3.534, 5.105)$ or degrees $(22.5°, 112.5°, 202.5°, 292.5°)$ are given, but all 4 solutions are found, penalise one A mark only.<br>Ignore solutions out of range. | **[12]** |

**January 2007**

| Question Number | Scheme | Marks |
|---|---|---|
| 1. | (a) $\sin 3\theta = \sin(2\theta+\theta) = \sin 2\theta\cos\theta + \cos 2\theta\sin\theta$ | B1 |
| | $= 2\sin\theta\cos^2\theta + (1-2\sin^2\theta)\sin\theta$ | B1 B1 |
| | $= 2\sin\theta - 2\sin^3\theta + \sin\theta - 2\sin^3\theta$ | M1 |
| | $= 3\sin\theta - 4\sin^3\theta$ * cso | A1 **(5)** |
| | (b) $\sin 3\theta = 3\times\frac{\sqrt{3}}{4} - 4\left(\frac{\sqrt{3}}{4}\right)^3 = \frac{3\sqrt{3}}{4} - \frac{3\sqrt{3}}{16} = \frac{9\sqrt{3}}{16}$ or exact equivalent | M1 A1 **(2)** |
| | | **[7]** |

**January 2008**

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** | (a) $\cos(2x+x) = \cos 2x\cos x - \sin 2x\sin x$ | M1 |
| | $= (2\cos^2 x - 1)\cos x - (2\sin x\cos x)\sin x$ | M1 |
| | $= (2\cos^2 x - 1)\cos x - 2(1-\cos^2 x)\cos x$ any correct expression | A1 |
| | $= 4\cos^3 x - 3\cos x$ | A1 (4) |
| | (b)(i) $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{\cos^2 x + (1+\sin x)^2}{(1+\sin x)\cos x}$ | M1 |
| | $= \frac{\cos^2 x + 1 + 2\sin x + \sin^2 x}{(1+\sin x)\cos x}$ | A1 |
| | $= \frac{2(1+\sin x)}{(1+\sin x)\cos x}$ | M1 |
| | $= \frac{2}{\cos x} = 2\sec x$ * cso | A1 (4) |
| | (c) $\sec x = 2$ *or* $\cos x = \frac{1}{2}$ | M1 |
| | $x = \frac{\pi}{3}, \frac{5\pi}{3}$ accept awrt 1.05, 5.24 | A1, A1 (3) |
| | | **[11]** |

| Question Number | **January 2009** Scheme | Marks |
|---|---|---|
| **6**(a) (i) | $\sin 3\theta = \sin(2\theta + \theta)$ | |
| | $= \sin 2\theta \cos\theta + \cos 2\theta \sin\theta$ | |
| | $= 2\sin\theta\cos\theta.\cos\theta + (1 - 2\sin^2\theta)\sin\theta$ | M1 A1 |
| | $= 2\sin\theta(1 - \sin^2\theta) + \sin\theta - 2\sin^3\theta$ | M1 |
| | $= 3\sin\theta - 4\sin^3\theta$ ✱ cso | A1 (4) |
| (ii) | $8\sin^3\theta - 6\sin\theta + 1 = 0$ | |
| | $-2\sin 3\theta + 1 = 0$ | M1 A1 |
| | $\sin 3\theta = \frac{1}{2}$ | M1 |
| | $3\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ | |
| | $\theta = \frac{\pi}{18}, \frac{5\pi}{18}$ | A1 A1 (5) |
| (b) | $\sin 15° = \sin(60° - 45°) = \sin 60° \cos 45° - \cos 60° \sin 45°$ | M1 |
| | $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ | M1 A1 |
| | $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ✱ cso | A1 (4) |
| | *Alternatives to* (b) | |
| | ① $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$ | M1 |
| | $= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ | M1 A1 |
| | $= \frac{1}{4}\sqrt{6} - \frac{1}{4}\sqrt{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ✱ cso | A1 (4) |
| | ② Using $\cos 2\theta = 1 - 2\sin^2\theta$, $\cos 30° = 1 - 2\sin^2 15°$ | |
| | $2\sin^2 15° = 1 - \cos 30° = 1 - \frac{\sqrt{3}}{2}$ | |
| | $\sin^2 15° = \frac{2 - \sqrt{3}}{4}$ | M1 A1 |
| | $\left(\frac{1}{4}(\sqrt{6} - \sqrt{2})\right)^2 = \frac{1}{16}(6 + 2 - 2\sqrt{12}) = \frac{2 - \sqrt{3}}{4}$ | M1 |
| | Hence $\sin 15° = \frac{1}{4}(\sqrt{6} - \sqrt{2})$ ✱ cso | A1 (4) |

**January 2010**

| Question Number | Scheme | Marks |
|---|---|---|
| Q8 | $\text{cosec}^2 2x - \cot 2x = 1, \quad (\text{eqn } *) \qquad 0 \le x \le 180^\circ$ | |
| | Using $\text{cosec}^2 2x = 1 + \cot^2 2x$ gives<br>$1 + \cot^2 2x - \cot 2x = 1$ | M1 |
| | $\underline{\cot^2 2x - \cot 2x} = 0$ or $\cot^2 2x = \cot 2x$ | A1 |
| | $\cot 2x(\cot 2x - 1) = 0$ or $\cot 2x = 1$ | dM1 |
| | $\cot 2x = 0$ or $\cot 2x = 1$ | A1 |
| | $\cot 2x = 0 \Rightarrow (\tan 2x \to \infty) \Rightarrow 2x = 90, 270$<br>$\Rightarrow x = 45, 135$<br>$\cot 2x = 1 \Rightarrow \tan 2x = 1 \Rightarrow 2x = 45, 225$<br>$\Rightarrow x = 22.5, 112.5$ | M1 |
| | Overall, $x = \{22.5, 45, 112.5, 135\}$ | A1<br>B1 |
| | | **[7]** |

**June 2010**

| Question Number | Scheme | Marks |
|---|---|---|
| **1.** (a) | $\dfrac{2\sin\theta\cos\theta}{1 + 2\cos^2\theta - 1}$ | M1 |
| | $\dfrac{\not{2}\sin\theta \not{\cos\theta}}{\not{2}\cos\theta\not{\cos\theta}} = \tan\theta$ (as required) **AG** | A1 **cso** |
| | | (2) |
| (b) | $2\tan\theta = 1 \Rightarrow \tan\theta = \dfrac{1}{2}$ | M1 |
| | $\theta_1 = \text{awrt } 26.6^\circ$ | A1 |
| | $\theta_2 = \text{awrt } -153.4^\circ$ | A1 $\sqrt{}$ |
| | | (3) |
| | | **[5]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3.** | $2\cos 2\theta = 1 - 2\sin\theta$ | | |
| | $2\left(1 - 2\sin^2\theta\right) = 1 - 2\sin\theta$ | Substitutes either $1 - 2\sin^2\theta$ or $2\cos^2\theta - 1$ or $\cos^2\theta - \sin^2\theta$ for $\cos 2\theta$. | M1 |
| | $2 - 4\sin^2\theta = 1 - 2\sin\theta$ | | |
| | $4\sin^2\theta - 2\sin\theta - 1 = 0$ | Forms a "quadratic in sine" = 0 | M1(*) |
| | $\sin\theta = \frac{2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ | Applies the quadratic formula<br>See notes for alternative methods. | M1 |
| | PVs: $\alpha_1 = 54^\circ$ or $\alpha_2 = -18^\circ$ | | |
| | | Any one correct answer | A1 |
| | $\theta = \{54, 126, 198, 342\}$ | 180-their pv | dM1(*) |
| | | All four solutions correct. | A1 |
| | | | **[6]** |

**June 2011**

| Question Number | Scheme | | Marks |
|---|---|---|---|
| 6 (a) | $\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \frac{1-\cos 2\theta}{\sin 2\theta}$ | | M1 |
| | $= \frac{2\sin^2\theta}{2\sin\theta\cos\theta}$ | | M1A1 |
| | $= \frac{\sin\theta}{\cos\theta} = \tan\theta$ | cso | A1* (4) |
| (b)(i) | $\tan 15° = \frac{1}{\sin 30°} - \frac{\cos 30°}{\sin 30°}$ | | M1 |
| | $\tan 15° = \frac{1}{\frac{1}{2}} - \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2 - \sqrt{3}$ | cso | dM1 A1* (3) |
| (b)(ii) | $\tan 2x = 1$ | | M1 |
| | $2x = 45°$ | | A1 |
| | $2x = 45° + 180°$ | | M1 |
| | $x = 22.5°, 112.5°, 202.5°, 292.5°$ | | A1(any two) A1 (5) |
| | | | 12 Marks |
| | Alt for (b)(i) $\tan 15° = \tan(60° - 45°)$ *or* $\tan(45° - 30°)$ | | |
| | $\tan 15° = \frac{\tan 60 - \tan 45}{1 + \tan 60 \tan 45}$ *or* $\frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30}$ | | M1 |
| | $\tan 15° = \frac{\sqrt{3} - 1}{1 + \sqrt{3}}$ *or* $\frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{\sqrt{3}}{3}}$ | | M1 |
| | Rationalises to produce $\tan 15° = 2 - \sqrt{3}$ | | A1* |

| | | |
|---|---|---|
| **5.** | **Uses the identity** $\cot^2(3\theta) = \text{cosec}^2(3\theta) - 1$ *in* | M1 |
| | $2\cot^2(3\theta) = 7\text{cosec}(3\theta) - 5$ | |
| | $2\text{cosec}^2(3\theta) - 7\text{cosec}(3\theta) + 3 = 0$ | A1 |
| | $(2\text{cosec}3\theta - 1)(\text{cosec}3\theta - 3) = 0$ | M1 |
| | $\text{cosec}3\theta = 3$ | A1 |
| | $\theta = \dfrac{invsin\left(\frac{1}{3}\right)}{3}, \quad \dfrac{19.5°}{3} = awrt\ 6.5°$ | M1, A1 |
| | $\theta = \dfrac{180° - invsin\left(\frac{1}{3}\right)}{3}, 53.5°$ Correct 2nd value | M1,A1 |
| | $\theta = \dfrac{360° + invsin\left(\frac{1}{3}\right)}{3}$ | M1 |
| | Correct 3rd value | |
| | All 4 correct answers awrt 6.5°,53.5°,126.5° or 173.5° | **A1** |
| | | **(10 marks)** |

**June 2012**

| Question Number | Scheme | Marks |
|---|---|---|
| **8.** (a) | $R=25$ | B1 |
| | $\tan\alpha = \frac{24}{7} \Rightarrow \alpha = \text{(awrt)}73.7^0$ | M1 A1 |
| | | (3) |
| (b) | $\cos(2x + their\ \alpha) = \frac{12.5}{their\ R}$ | M1 |
| | $2x + their\ '\alpha' = 60^0$ | A1 |
| | $2x + their\ '\alpha' = \text{their } 300^0 \text{ or their } 420^0 \Rightarrow x = ..$ | M1 |
| | $x = awrt\ 113.1^0, 173.1^0$ | A1 A1 |
| | | (5) |
| (c) | Attempts to use $\cos 2x = 2\cos^2 x - 1$ **AND** $\sin 2x = 2\sin x \cos x$ in the expression | M1 |
| | $14\cos^2 x - 48\sin x\cos x = 7(\cos 2x + 1) - 24\sin 2x$ | |
| | $= 7\cos 2x - 24\sin 2x + 7$ | A1 |
| | | (2) |
| (d) | $14\cos^2 x - 48\sin x\cos x = R\cos(2x + \alpha) + 7$ | |
| | Maximum value =’*R*’+’*c*’ | M1 |
| | = 32 cao | A1 |
| | | (2) |
| | | **(12 marks)** |

| Question Number | Scheme | Marks |
|---|---|---|
| **6.** | (i) $(\sin 22.5+\cos 22.5)^2 = \sin^2 22.5+\cos^2 22.5+......$ | M1 |
| | $= \sin^2 22.5+\cos^2 22.5+2\sin 22.5\cos 22.5$ | |
| | States or uses $\sin^2 22.5+\cos^2 22.5=1$ | B1 |
| | **Uses** $2\sin x\cos x=\sin 2x \Rightarrow 2\sin 22.5\cos 22.5=\sin 45$ | M1 |
| | $(\sin 22.5+\cos 22.5)^2 = 1+\sin 45$ | A1 |
| | $=1+\frac{\sqrt{2}}{2}$ or $1+\frac{1}{\sqrt{2}}$ cso | A1 |
| | | (5) |
| | (ii) (a) $\cos 2\theta+\sin\theta=1 \Rightarrow 1-2\sin^2\theta+\sin\theta=1$ | M1 |
| | $\sin\theta-2\sin^2\theta=0$ | |
| | $2\sin^2\theta-\sin\theta=0$ or $k=2$ | A1* |
| | | (2) |
| | (b) $\sin\theta(2\sin\theta-1)=0$ | M1 |
| | $\sin\theta=0,\quad \sin\theta=\frac{1}{2}$ | A1 |
| | Any two of 0,30,150,180 | B1 |
| | All four answers 0,30,150,180 | A1 |
| | | (4) |
| | | **(11 marks)** |

**June 2013**

| **Question Number** | **Scheme** | **Marks** |
|---|---|---|
| **6.(i)** | $\text{cosec}2x = \dfrac{1}{\sin 2x}$ | M1 |
| | $= \dfrac{1}{2\sin x\cos x}$ | M1 |
| | $= \dfrac{1}{2}\text{cosec}x\sec x \Rightarrow \quad \lambda = \dfrac{1}{2}$ | A1 |
| | | **(3)** |
| **(ii)** | $3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$ | M1 |
| | $\sec^2\theta + 3\sec\theta + 2 = 0$ | |
| | $(\sec\theta + 2)(\sec\theta + 1) = 0$ | M1 |
| | $\sec\theta = -2, -1$ | A1 |
| | $\cos\theta = -0.5, -1$ | M1 |
| | $\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$ | A1A1 |
| | | **(6)** |
| | | **[9]** |

| Question Number | Scheme | | Marks |
|---|---|---|---|
| **3. (i) (a)** | $2\frac{\sin x}{\cos x} - \frac{\cos x}{\sin x} = \frac{5}{\sin x}$ | | B1 |
| | Uses common denominator to give $2\sin^2 x - \cos^2 x = 5\cos x$ | | M1 |
| | Replaces $\sin^2 x$ by $(1-\cos^2 x)$ to give $2(1-\cos^2 x) - \cos^2 x = 5\cos x$ | | M1 |
| | Obtains $3\cos^2 x + 5\cos x - 2 = 0$ ( $a$ =3, $b$ = 5, $c$ = -2 ) | | A1 (4) |
| **(b)** | Solves $3\cos^2 x + 5\cos x - 2 = 0$ to give cos $x$ = | | M1 |
| | cos $x$ = $\frac{1}{3}$ only (rejects cos $x$ = -2) | | A1 |
| | So $x$ = 1.23 or 5.05 | | dM1A1 (4) |
| **(ii)** | Either | Or | |
| | $\tan\theta + \cot\theta \equiv \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$ | $\tan\theta + \cot\theta \equiv \tan\theta + \frac{1}{\tan\theta}$ | B1 |
| | $\equiv \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$ | $\equiv \frac{\tan^2\theta + 1}{\tan\theta}$ | M1 |
| | $\equiv \frac{2}{\sin 2\theta}$ | $\equiv \frac{1}{\cos^2\theta \times \frac{\sin\theta}{\cos\theta}} \equiv \frac{2}{\sin 2\theta}$ | M1 |
| | $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda = 2$) | $\equiv 2\operatorname{cosec}2\theta$ (so $\lambda = 2$) | A1 (4) |
| | | | **12 marks** |

**June 2015**

| Question Number | Scheme | Marks |
|---|---|---|
| **1.(a)** | $\tan 2\theta° = \dfrac{2\tan\theta°}{1-\tan^2\theta°} = \dfrac{2p}{1-p^2}$ | M1A1 |
| | | **(2)** |
| **(b)** | $\cos\theta° = \dfrac{1}{\sec\theta°} = \dfrac{1}{\sqrt{1+\tan^2\theta°}} = \dfrac{1}{\sqrt{1+p^2}}$ | M1A1 |
| | | **(2)** |
| **(c)** | $\cot(\theta-45)° = \dfrac{1}{\tan(\theta-45)°} = \dfrac{1+\tan\theta°\tan 45°}{\tan\theta°-\tan 45°} = \dfrac{1+p}{p-1}$ | M1A1 |
| | | **(2)** |
| | | **(6 marks)** |

**June 2016**

| **Question** | **Scheme** | **Marks** |
|---|---|---|
| **8.** (a) | $2\cot 2x + \tan x \equiv \dfrac{2}{\tan 2x} + \tan x$ | B1 |
| | $\equiv \dfrac{(1-\tan^2 x)}{\tan x} + \dfrac{\tan^2 x}{\tan x}$ | M1 |
| | $\equiv \dfrac{1}{\tan x}$ | M1 |
| | $\equiv \cot x$ | A1* |
| | | **(4)** |
| (b) | $6\cot 2x + 3\tan x = \operatorname{cosec}^2 x - 2 \Rightarrow 3\cot x = \operatorname{cosec}^2 x - 2$ | |
| | $\Rightarrow 3\cot x = 1 + \cot^2 x - 2$ | M1 |
| | $\Rightarrow 0 = \cot^2 x - 3\cot x - 1$ | A1 |
| | $\Rightarrow \cot x = \dfrac{3 \pm \sqrt{13}}{2}$ | M1 |
| | $\Rightarrow \tan x = \dfrac{2}{3 \pm \sqrt{13}} \Rightarrow x = ..$ | M1 |
| | $\Rightarrow x = 0.294, -2.848, -1.277, 1.865$ | A2,1,0 |
| | | **(6)** |
| | | **(10 marks)** |