Q1

(a) $$\frac{\cos 2A}{\cos A + \sin A} = \frac{\cos^2 A - \sin^2 A}{\cos A + \sin A}$$

part of diff of two squares

$$= \frac{\cancel{(\cos A + \sin A)}(\cos A - \sin A)}{\cancel{(\cos A + \sin A)}}$$

$$= \cos A - \sin A = RHS$$

note order

(b) $$2 \operatorname{cosec} 2A \sin(B - A) = \frac{2}{\sin 2A} \cdot \sin B \cos A - \cos B \sin A$$

$$= \frac{\cancel{2}(\sin B \cos A - \cos B \sin A)}{\cancel{2} \sin A \cos A}$$

split fraction if you need to

$$\frac{\sin B \cancel{\cos A}}{\sin A \cancel{\cos A}} - \frac{\cos B \cancel{\sin A}}{\cancel{\sin A} \cos A} = LHS$$

(c) $$\frac{1 - \cos 2\theta}{\sin 2\theta} = \frac{1 - (1 - 2\sin^2\theta)}{2 \sin\theta \cos\theta}$$

aim to cancel $\sin\theta$

$$= \frac{\cancel{2} \sin^{\cancel{2}}\theta}{\cancel{2} \cancel{\sin\theta} \cos\theta} = \tan\theta = RHS$$

basic trig

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Q1

*(d) $\dfrac{\sec^2\theta}{1-\tan^2\theta} \equiv \dfrac{1}{\cos^2\theta(1-\tan^2\theta)} \equiv \dfrac{1}{\cos^2\theta - \cos^2\theta\tan^2\theta}$

$\cos\theta\tan\theta = \sin\theta$
$\cos^2\theta\tan^2\theta = \sin^2\theta$

$\equiv \dfrac{1}{\cos^2 - \sin^2\theta} \equiv \dfrac{1}{\cos 2\theta} \equiv \sec 2\theta = RHS$

(e) $2(\sin^3\theta\cos\theta + \cos^3\theta\sin\theta)$

$\sin^2\theta + \cos^2\theta = 1$
factorise

$\equiv 2(\sin\theta\cos\theta[\sin^2\theta + \cos^2\theta]) \equiv 2\sin\theta\cos\theta = RHS$
$\equiv 2\sin 2\theta$

create
common denominator

*(f) $\dfrac{\sin 3\theta}{\sin\theta} - \dfrac{\cos 3\theta}{\cos\theta} \equiv \dfrac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{\sin(3\theta - \theta)}{\sin\theta\cos\theta}$

now clear

$\equiv \dfrac{\sin 2\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{2\cancel{\sin\theta\cos\theta}}{\cancel{\sin\theta\cos\theta}} = RHS$

Q1

(g) $\operatorname{cosec}\theta - 2\cot 2\theta \cos\theta$

$\cot 2\theta = \dfrac{\cos 2\theta}{\sin 2\theta}$

$$\frac{1}{\sin\theta} - \frac{2\cos 2\theta}{\sin 2\theta}\cdot\cos\theta$$

$$\frac{1}{\sin\theta} - \frac{\cancel{2}\cos 2\theta \cancel{\cos\theta}}{\cancel{2}\sin\theta\cancel{\cos\theta}}$$

$$\frac{1}{\sin\theta} - \frac{1-2\sin^2\theta}{\sin\theta}$$

common denominator

$$\frac{2\sin^2\theta}{\sin\theta} = 2\sin\theta \equiv RHS$$

* (h) $$\frac{\sec\theta - 1}{\sec\theta + 1} \equiv \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$$

multiply by $\cos\theta$

$$\equiv \frac{1-\cos\theta}{1+\cos\theta}$$

$\theta = 2 \times \frac{\theta}{2}$

(if $A = \frac{\theta}{2}$ then $2A = \theta$)

$$\equiv \frac{1-\left(1-2\sin^2\frac{\theta}{2}\right)}{1+\left(1-2\sin^2\frac{\theta}{2}\right)}$$

A lovely twist

$$\equiv \frac{2\sin^2\frac{\theta}{2}}{2-2\sin^2\frac{\theta}{2}}$$

$$\equiv \frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}} \equiv \tan^2\frac{\theta}{2} \equiv RHS$$

Q1

* (i) $\tan\left(\frac{\pi}{4} - x\right) \equiv \dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\tan x}$ $\qquad \tan\frac{\pi}{4} = 1$

$\equiv \dfrac{1 - \tan x}{1 + \tan x}$

Need $\tan x \cos x = \sin x$ multiply top and bottom by $\cos x$

$\equiv \dfrac{\cos x - \sin x}{\cos x + \sin x}$

Difference of two squares again

$\equiv \dfrac{(\cos x - \sin x)}{(\cos x + \sin x)} \times \dfrac{(\cos x - \sin x)}{(\cos x - \sin x)}$

$\equiv \dfrac{\cos^2 x + \sin^2 x - 2\cos x \sin x}{\cos^2 x - \sin^2 x}$ $\qquad$ tidy up

$\equiv \dfrac{1 - 2\cos x \sin x}{\cos^2 x - \sin^2 x} \equiv \dfrac{1 - \sin 2x}{\cos 2x} \equiv \text{RHS}$

Q2

(a) $\sin(A+60°) + \sin(A-60°)$

$\equiv \left(\sin A \cos 60° + \cos A \sin 60°\right) + \left(\sin A \cos 60° - \cos A \sin 60°\right)$

$\equiv \left(\frac{1}{2}\sin A + \cancel{\frac{\sqrt{3}}{2}\cos A}\right) + \left(\frac{1}{2}\sin A - \cancel{\frac{\sqrt{3}}{2}\cos A}\right)$

$\equiv \sin A \equiv \text{RHS}$

(b) $\frac{\cos A}{\sin B} - \frac{\sin A}{\cos B} \equiv \frac{\cos A \cos B - \sin A \sin B}{\sin B \cos B}$ create common denominator

$\equiv \frac{\cos(A+B)}{\sin B \cos B} \equiv \text{RHS}$

(c) $\frac{\sin(x+y)}{\cos x \cos y} \equiv \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y}$

splitting creates clarity

$\equiv \frac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} + \frac{\cancel{\cos x} \sin y}{\cancel{\cos x} \cos y}$

$\equiv \tan x + \tan y \equiv \text{RHS}$

(d) $\frac{\cos(x+y)}{\sin x \sin y} + 1 \equiv \frac{\cos x \cos y - \sin x \sin y}{\sin x \sin y} + 1$

splitting may help

$\equiv \frac{\cos x \cos y}{\sin x \sin y} - \frac{\cancel{\sin x \sin y}}{\cancel{\sin x \sin y}} 1 + 1$

$\equiv \frac{\cos x}{\sin x} \frac{\cos y}{\sin y} \equiv \cot x \cot y \equiv \text{RHS}$

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Q2

(e) $\cos\left(\theta + \frac{\pi}{3}\right) + \sqrt{3}\sin\theta$

$\equiv \cos\theta\cos\frac{\pi}{3} - \sin\theta\sin\frac{\pi}{3} + \sqrt{3}\sin\theta$

$\equiv \frac{1}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta + \sqrt{3}\sin\theta$

We know

$\equiv \frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta$

$\frac{1}{2} = \sin\frac{\pi}{6}$

$\frac{\sqrt{3}}{2} = \cos\frac{\pi}{6}$

$\equiv \cos\theta\sin\frac{\pi}{6} + \sin\theta\cos\frac{\pi}{6}$

$\equiv \sin\left(\theta + \frac{\pi}{6}\right) \equiv$ RHS

* (f) $\cot(A+B) \equiv \frac{\cos(A+B)}{\sin(A+B)}$

$\equiv \frac{\cos A\cos B - \sin A\sin B}{\sin A\cos B + \cos A\sin B}$

splitting may help

$\equiv \frac{\cos A\cos B}{\sin A\cos B + \cos A\sin B} - \frac{\sin A\sin B}{\sin A\cos B + \cos A\sin B}$

$\div \sin A\sin B$

$\equiv \frac{\cot A\cot B}{\cot B + \cot A} - \frac{1}{\cot B + \cot A}$

$\equiv \frac{\cot A\cot B - 1}{\cot A + \cot B} \equiv$ RHS

Q2

(g) $\sin^2(45+\theta) + \sin^2(45-\theta)$

$\equiv [\sin(45+\theta)][\sin(45+\theta)] + [\sin(45-\theta)][\sin(45-\theta)]$

$\equiv (\sin 45 \cos\theta + \cos 45 \sin\theta)^2 + (\sin 45 \cos\theta - \cos 45 \sin\theta)^2$

$\equiv \left(\frac{\cos\theta}{\sqrt{2}} + \frac{\sin\theta}{\sqrt{2}}\right)^2 + \left(\frac{\cos\theta}{\sqrt{2}} - \frac{\sin\theta}{\sqrt{2}}\right)^2$

$\equiv \frac{\cos^2\theta}{2} + \frac{\sin^2\theta}{2} + \cancel{\frac{2\sin\theta\cos\theta}{2}} + \frac{\cos^2\theta}{2} + \frac{\sin^2\theta}{2} - \cancel{\frac{2\sin\theta\cos\theta}{2}}$

$\equiv \cos^2\theta + \sin^2\theta \equiv 1 \equiv$ RHS

** (h) $\cos(A+B)\cos(A-B) \equiv (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

Difference of two squares $\equiv (\cos A\cos B)^2 - (\sin A\sin B)^2$

$\equiv \cos^2 A\cos^2 B - \sin^2 A\sin^2 B$

Try identity using $\sin^2\theta + \cos^2\theta = 1$ to find alternatives

$\equiv \cos^2 A(1-\sin^2 B) - \sin^2 B(1-\cos^2 A)$

$\equiv \cos^2 A - \cancel{\cos^2 A\sin^2 B} - \sin^2 B + \cancel{\sin^2 B\cos^2 A}$

$\equiv \cos^2 A - \sin^2 B \equiv$ RHS

Q3

(a) $2\operatorname{cosec} 2\theta \equiv \dfrac{2}{\sin 2\theta} \equiv \dfrac{2}{2\sin\theta\cos\theta}$

$\sin^2\theta + \cos^2\theta \equiv 1$

$\equiv \dfrac{1}{\sin\theta\cos\theta} \equiv \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta}$

$\equiv \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} \equiv \tan\theta + \cot\theta \equiv LHS$

(b) $\tan 75 + \cot 75 = 2\operatorname{cosec} 150$

$2\operatorname{cosec} 150 = \dfrac{2}{\sin 150} = \dfrac{2}{\sin 30}$

$= \dfrac{2}{\frac{1}{2}} = 4$

Q4 *

(a) $\sin 3\theta \equiv \sin(\theta + 2\theta) \equiv \sin\theta\cos 2\theta + \cos\theta\sin 2\theta$

choose the right one

$\equiv \sin\theta(\cos^2\theta - \sin^2\theta) + \cos\theta\, 2\sin\theta\cos\theta$

$\equiv \sin\theta\cos^2\theta - \sin^3\theta + 2\sin\theta\cos^2\theta$

$\equiv 3\sin\theta\cos^2\theta - \sin^3\theta$

(b) $\cos 3\theta \equiv \cos(\theta + 2\theta)$

$\equiv \cos\theta\cos 2\theta - \sin\theta\sin 2\theta$ and deal with 2θ

$\equiv \cos\theta(\cos^2\theta - \sin^2\theta) - 2\sin\theta\sin\theta\cos\theta$

$\equiv \cos^3\theta - \sin^2\theta\cos\theta - 2\sin^2\theta\cos\theta$

$\equiv \cos^3\theta - 3\sin^2\theta\cos\theta \equiv \text{RHS}$

Q5 a(i) $2\cos^2\frac{x}{2} - 1 \equiv \cos x$

$2\cos^2\frac{x}{2} \equiv \cos x + 1$

$\cos^2\frac{x}{2} \equiv \frac{\cos x + 1}{2}$

a(ii)

$1 - 2\sin^2\frac{x}{2} \equiv \cos x$

$1 - \cos x \equiv 2\sin^2\frac{x}{2}$

$\frac{1 - \cos x}{2} = \sin^2 x$

Q5

(b) (i) $\cos^2\frac{\theta}{2} = \frac{1+0.6}{2} = 0.8 = \frac{4}{5}$

$\cos\frac{\theta}{2} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

(ii) $\sin^2\frac{\theta}{2} = \frac{1-0.6}{2} = \frac{0.4}{2} = 0.2 = \frac{1}{5}$

$\sin\frac{\theta}{2} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$

(iii) $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{\frac{\sqrt{5}}{\cancel{5}}}{\frac{2\sqrt{5}}{\cancel{5}}} = \frac{1}{2}$

(c)* $\cos^4\frac{A}{2} \equiv \left(\cos^2\frac{A}{2}\right)^2$

$\equiv \left(\frac{1+\cos A}{2}\right)^2 \equiv \frac{1+2\cos A+\cos^2 A}{4}$ And again

$\equiv \frac{1+2\cos A+\frac{1+\cos 2A}{2}}{4}$ mult all by 2

$\equiv \frac{2+4\cos A+1+\cos 2A}{8}$

$\equiv \frac{3+4\cos A+\cos 2A}{8} \equiv \frac{1}{8}(3+4\cos A+\cos 2A)$

$\equiv$ RHS

(10)