C4 Cheat Sheet

Chapter	Usual types of questions	Tips	What can go ugly
1 – Partial Fractions	• Be able to split a fraction whose denominator is a product of linear expressions, e.g. $\frac{2x+3}{x(x+1)}$ • Be able to split a fraction where one (or more) of the factors in the denominator are squared, e.g. $\frac{2x+3}{x^2(x+1)}$ • Deal with top-heavy fractions where the highest power in the denominator is greater or equal to the highest power in the denominator, e.g $\frac{x^2+2}{x(x+1)}$	• The textbook provides two methods for dealing with top heavy fractions. The algebraic long division method is miles easier! e.g. $\frac{x^2+2}{x(x+1)} = \frac{x^2+2}{x^2+x}$. Using long division we get a quotient of 1 and a remainder of $-x+2$, thus: $$\frac{x^2+2}{x^2+x} = 1 + \frac{-x+2}{x(x+1)}$$ Then split the $\frac{-x+2}{x(x+1)}$ into partial fractions as normal. • Don't forget that when you have a squared factor in the denominator, you need two fractions in your partial fraction sum: $$\frac{2}{x^2(x+1)} \equiv \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$ • When you have three unknowns it's generally easiest to use substitution to get two of them (e.g. the A and the B) then compare the coefficients of x^2 to get the C. For the above example: $$2 \equiv Ax(x+1) + B(x+1) + Cx^2$$ We can see immediately, without needing to write out the expansion, that $0 = A + C$, by comparing x^2 terms.	• Forgetting the extra term when the denominator's factors are squared. • Being sloppy at algebraic long division! • Be careful with substitution of negative values. • You may have to spot that you need to factorise the denominator first before expressing as partial fractions. • Not realising the fraction is top heavy and therefore trying to incorrectly do: $$\frac{2x^2}{x(x+1)} = \frac{A}{x} + \frac{B}{x+1}$$
2 – Parametric Equations	• Know that $\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$ (This makes sense as we have just divided numerator and denominator by dt) • Be able to integrate parametric equations. • Be able to convert parametric equations into a single Cartesian one.	• Note: You will NOT be asked to sketch parametric equations. • To convert parametric equations involving trig functions to Cartesian ones, the strategy is usually to make $\sin x$ and $\cos x$ the subject before using the identity $\sin^2 x + \cos^2 x \equiv 1$. Often squaring one of the parametric equations helps so that we have $\sin^2 x$ and/or $\cos^2 x$: $x = \sqrt{3}\sin 2t \qquad y = 4\cos^2 t$ $x = 2\sqrt{3}\sin t\ \cos t$ $x^2 = 12\sin^2 t \cos^2 t$ $x^2 = 12(1-\cos^2 t)\cos^2 t$ $x^2 = 12\left(1-\left(\frac{y}{4}\right)^2\right)\frac{y}{4}$	• Hitting a dead end converting parametric equations to Cartesian. See tips on left. • Forgetting to multiply by $\frac{dx}{dt}$ when integrating parametric equations. Remember that the dx in $\int y\ dx$ can be replaced with $\frac{dx}{dt}\ dt$, which is easy to remember, as the dt's cancel if we think of dx and dt just as quantities.

3 – Binomial Expansion	• Expanding out an expression of the form $(1+kx)^n$, where n is negative or fractional. • Expanding out an expression of the form $(a+kx)^n$, where a needs to be factorised out first. • Finding the product of two Binomial expansions, e.g. $\frac{\sqrt{1+x}}{\sqrt{1-x}} \to (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$	• $(1+kx)^n = 1 + n(kx) + \frac{n(n-1)}{2!}(kx)^2 + \frac{n(n-1)(n-2)}{3!}(kx)^3 + \cdots$ • Your expression may be a binomial expansion in disguise, e.g. $\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}} = 1 + \left(-\frac{1}{2}\right)(-2x) + \frac{-\frac{1}{2} \times -\frac{3}{2}}{2!}(-2x)^2 + \cdots$ • When the first term is not 1, you have to factorise this number out, raised to the power outside the brackets. e.g. $(4+5x)^{\frac{1}{2}} = 4^{\frac{1}{2}}\left(1+\frac{5}{4}x\right)^{\frac{1}{2}}$ $= 2\left[1 + \frac{1}{2}\left(\frac{5}{4}x\right) + \cdots\right]$ Ensure the outer brackets are maintained till the very end, when you expand them out. • When finding the product of two expansions, then if you needed up to the x^2 term, then you only need to find up to the x^2 term in each of the two expansions. Only consider things in the expansion which are up to x^2. e.g. $\sqrt{\frac{1+x}{1-x}} = \frac{\sqrt{1+x}}{\sqrt{1-x}} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$ $(1+x)^{\frac{1}{2}} \approx 1 + \frac{1}{2}x - \frac{1}{8}x^2$ $(1-x)^{-\frac{1}{2}} \approx 1 + \frac{1}{2}x + \frac{3}{8}x^2$ $(1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}} \approx \left(1+\frac{1}{2}x-\frac{1}{8}x^2\right)\left(1+\frac{1}{2}x+\frac{3}{8}x^2\right)$ $= 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{1}{2}x + \frac{1}{4}x^2 - \frac{1}{8}x^2$ $= 1 + x + \frac{1}{2}x^2$	Many things! • Lack of brackets when squaring/cubing things, e.g. you need $(2x)^3 = 8x^3$ not $2x^3$ • With say $(3+4x)^{-1}$, forgetting to raise the 3 you factor out to the power of -1. • Forgetting to put the factorial in the denominators of the Binomial coefficients (a common error is $\frac{\square}{3}$ instead of $\frac{\square}{3!}$) • Being careless in using your calculator when simplifying coefficients. • Be ridiculously careful with signs! • Accidentally forgetting the minus in the power when expanding say $\frac{1}{(x+1)^2}$

4 - Differentiation	• Appreciate that $y = a^x$ represents 'exponential growth' when $a > 1$, and 'exponential decay' when $0 < a < 1$ (and from C3, know the graphs for each). • Know that $\frac{d}{dx}(a^x) = a^x \ln a$ (proof unlikely to be asked for) • Be able to differentiate implicitly, e.g. $\frac{d}{dx}(y^2) = 2y\frac{dy}{dx}$ and subsequently be able to make $\frac{dy}{dx}$ the subject. • Be able to set up differential equations, e.g. understand that "the temperature falls at a rate proportional to its current temperature" could be represented as $\frac{dT}{dt} = -kT$ • Connect different derivatives involving rates, e.g. $\frac{dA}{dx} = \frac{dA}{dt} \times \frac{dt}{dx}$	• Example of implicit differentiation (which involves collecting the $\frac{dy}{dx}$ terms on one side and factorising it out): "Given that $xy^2 + 2y = x^2$, find $\frac{dy}{dx}$." Differentiating both sides with respect to x: $x\left(2y\frac{dy}{dx}\right) + y^2 + 2\frac{dy}{dx} = 2x$ $\frac{dy}{dx}(2xy + 2) = 2x - y^2$ $\frac{dy}{dx} = \frac{2x - y^2}{2xy + 2}$ They particularly love use of the product rule! • A 'differential equation' is an equation involving both some variables and derivatives involving those variables, e.g. a mix of x, y and $\frac{dy}{dx}$. 'Solving' this equation means to obtain an equation only involving the variables, and not the derivatives. • Whenever you see the word 'rate', think $/dt$. • "A circle's radius increases at a rate of 2cm/s. Find the rate of increase of its area when the radius is 10cm." First note the variables involved: A, r and because we're talking about rates, t. We need to find $\frac{dA}{dt}$. Since derivatives behave pretty much like normal fractions, first write the following product with the dA and dt copied into the diagonals: $\frac{dA}{dt} = \frac{dA}{\square} \times \frac{\square}{dt}$ Then fill the remaining diagonals with the remaining variable, dr: $\frac{dA}{dt} = \frac{dA}{dr} \times \frac{dr}{dt}$ One value, in this case $\frac{dr}{dt} = 2$, is always given. The other we need to form some formula, in this case $A = \pi r^2$ (and often using simple geometry to find an area of volume), and differentiate: $\frac{dA}{dr} = 2\pi r$ Thus when $r = 10$, $\frac{dA}{dr} = 2 \times \pi \times 10 = 20\pi$ Thus: $\frac{dA}{dt} = 20\pi \times 2 = 40\pi$	• A classic is to accidentally treat x or y as constants rather than variables, when differentiating implicitly. Note that $\frac{d}{dx}(ax) = a$ if a is a constant, but $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$ by the product rule, and not just y. • When differentiating implicitly, you might forget to put the $\frac{dy}{dx}$, e.g. $\frac{d}{dx}(y^2) = 2y$ rather than the correct $2y\frac{dy}{dx}$ • Exponential functions do not behave like polynomials when differentiated. e.g. $\frac{d}{dx}(x^3) = 3x^2$, but $\frac{d}{dx}(3^x) = 3^x \ln 3$, and absolutely not $x \cdot 3^{x-1}$! • Many students often get their equation wrong when connecting rates of change, often say dividing instead of multiplying, or vice versa. If you use the 'fill in the diagonals' tip on the left this will unlikely be a problem.

<table>
<tr>
<td>5 - Vectors</td>
<td>(In rough descending order of how frequently they appear in exams)
• Find the point of intersection of two lines or prove that two lines do not intersect.
• Find the angle between two lines.
• Finding a missing $x/y/z$ value of a point on a line.
• Find the length of a vector or the distance between two points.
• Find the nearest point on a line to a point not on the line (often the origin) – note: not in your textbook!
• Show lines are perpendicular.
• Show a point lies on a line.
• Show 3 points are collinear (i.e. lie on the same straight line)
• Find the area of a rectangle, parallelogram or triangle formed by vectors.
• Find the equation of a line.
• Find the reflection of a point in a line.
• Find the point after going a specific distance in the direction of a given vector.</td>
<td>• When you see the i, j, k unit vectors used in an exam question, never actually use this notation yourself: always just write all vectors in conventional column vector form.
• Almost always draw a suitable diagram. This will be particularly helpful when you need to find the area of some shape (typically the last part of a question).
• When finding the area of a shape, you can almost always use your answers from previous parts of the questions, including lengths of vectors and angles between two vectors. Remember that area of non-right angled triangle $= \frac{1}{2}ab\sin C$ where the angle C appears between the two sides a and b. A parallelogram can be cut in half to form two congruent non-right angled triangles (i.e. multiply by 2).
• To show 3 points A, B, C are collinear, just show that $\overrightarrow{AB}$ is a multiple of $\overrightarrow{BC}$ (i.e. vectors are parallel).
• "Show $3\boldsymbol{i} + 3\boldsymbol{j} + 2\boldsymbol{k}$ lies on the line with vector equation $r = \boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k} + t(\boldsymbol{i} - \boldsymbol{k})$"
i.e. Show $\begin{pmatrix}3\\3\\2\end{pmatrix}$ lies on $\begin{pmatrix}1+t\\3\\4-t\end{pmatrix}$. Equating $3 = 1 + t$ to $t = 2$.
Then $4 - t = 4 - 2 = 2$, so y and z components are same.
• "Let $l_1: \boldsymbol{r} = (9\boldsymbol{i} + 13\boldsymbol{j} - 3\boldsymbol{k}) + \lambda(\boldsymbol{i} + 4\boldsymbol{j} - 2\boldsymbol{k})$
Given point A has position vector $4i + 16j - 3k$ and P lies on l_1 such that AP is perpendicular to l_1, find P."
$$l_1: \begin{pmatrix}9+\lambda\\13+4\lambda\\-3-2\lambda\end{pmatrix}$$
Note that the direction vector of the line, and the vector $\overrightarrow{PA}$ are perpendicular. P is just a point on the line so can be represented as $\begin{pmatrix}9+\lambda\\13+4\lambda\\-3-2\lambda\end{pmatrix}$ for some specific λ we need to find.
Direction vector of l_1 is $\begin{pmatrix}1\\4\\-2\end{pmatrix}$</td>
<td>• When finding the angle between two lines, accidentally using the full vector representation of the line (in your dot product), and not just the direction component, e.g. using $\begin{pmatrix}1-t\\2\\3+2t\end{pmatrix}$ instead of just the correct $\begin{pmatrix}-1\\0\\t\end{pmatrix}$.
• Making sign errors when subtracting vectors, particularly when subtracting an expression involving a negative. Correctly:
$$\begin{pmatrix}1\\2\\3\end{pmatrix} - \begin{pmatrix}1-t\\-2\\2t-4\end{pmatrix} = \begin{pmatrix}t\\4\\7-2t\end{pmatrix}$$
• Once finding out s and t (or μ and λ) when solving simultaneous equation to find the intersection of two lines, forgetting to show that these satisfy the remaining equation.
• Forgetting the square root when finding the magnitude of a vector.</td>
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		$\overrightarrow{PA} = \begin{pmatrix}4\\16\\-3\end{pmatrix} - \begin{pmatrix}9+\lambda\\13+4\lambda\\-3-2\lambda\end{pmatrix} = \begin{pmatrix}-5-\lambda\\3-4\lambda\\2\lambda\end{pmatrix}$ Thus: $\begin{pmatrix}-5-\lambda\\3-4\lambda\\2\lambda\end{pmatrix} \cdot \begin{pmatrix}1\\4\\-2\end{pmatrix} = 0$ $-5-\lambda+12-16\lambda-4\lambda=0$ $\lambda = \frac{1}{3}$ Thus: $P = \begin{pmatrix}9+\frac{1}{3}\\13+4(\frac{1}{3})\\-3-2(\frac{1}{3})\end{pmatrix} = \begin{pmatrix}9\frac{1}{3}\\14\frac{1}{3}\\-3\frac{2}{3}\end{pmatrix}$ • Suppose we wanted to start from a position vector $\begin{pmatrix}2\\2\\2\end{pmatrix}$ and wanted to go 10 units in the direction $\begin{pmatrix}1\\0\\3\end{pmatrix}$. The key is to convert the direction to a unit vector, because moving by it then means we've moved a distance of 1. Since $\sqrt{1^2+0^2+3^2}=2$, the direction as a unit vector is $\frac{1}{2}\begin{pmatrix}1\\0\\3\end{pmatrix}$. Thus the new point is $\begin{pmatrix}2\\2\\2\end{pmatrix} + 10 \times \frac{1}{2}\begin{pmatrix}1\\0\\3\end{pmatrix} = \begin{pmatrix}7\\2\\17\end{pmatrix}$	
6 - Integration	• Integrate a large variety of expressions. See the 'integration cheat sheet' overleaf. But by category: ○ Integrating trig functions, including reciprocal functions and squared functions $\sin^2 x$, $\cos^2 x$, $\sec^2 2x$, etc. ○ Integrating by 'reverse chain rule' (also known as 'integration by inspection'). ○ Integrating by a given substitution.	• One often forgotten integration is exponential functions such as 2^x. Differentiating has effect of multiplying by ln of the base, and thus integrating divides by it. i.e. $\frac{d}{dx}(2^x) = \ln 2 \cdot 2^x \qquad \int 2^x\,dx = \frac{1}{\ln 2}2^x + C$ • Know the two double angle formulae for cos like the back of your hand, for use when integrating $\sin^2 x$ or $\cos^2 x$ • In general, know your integrals of all the 'trig squares', i.e. $\sin^2 x, \cos^2 x, \tan^2 x, cosec^2 x, \sec^2 x, \cot^2 x$ • For integration by 'reverse chain rule', always 'consider' some sensible expression to differentiate, then adjust for the factor difference. e.g. $\int (4-3x)^5 dx$ Then your working might be: "Consider $y=(4-3x)^6$. Then	Where to start! • One big problem is just not knowing what method to use to integrate a particular expression. The cheat sheet overleaf should help, as should lots of practice of a variety of expressions! • Similarly getting stuck on integration by substitution, because you can't get the whole original expression only in terms of the new variable (t or otherwise). • Perhaps the all-time biggest mistake is forgetting to consider the effects of chain rule. e.g. Accidentally doing $\int \cos 2x\,dx = \sin 2x$

- Integration by parts.
- Integrating by use of partial fractions.
- Integrating top heavy fractions by algebraic division.

- Be able to differentiate parametric equations:
$$\int y\,dx = \int y\frac{dx}{dt}\,dt$$
- Calculate volumes of revolution both for normal and parametric equations:
$$V = \pi\int y^2\,dx$$
$$V = \pi\int y^2\frac{dx}{dt}\,dt$$
- Solve differential equations.
e.g. $\frac{dy}{dx} = xy + x$
- Trapezium Rule as per C2, but now with C3/C4 expressions to integrate. You will frequently be asked to compare the actual area and the estimated area using the rule, and the percentage error.

$$\frac{dy}{dx} = 6(4-3x)^5 \times (-3) = -18(4-3x)^5$$
$$\therefore \int (4-3x)^5 dx = -\frac{1}{18}(4-3x)^6 + C$$

- For integration by substitution, the official specification says "Except in the simplest of cases, the substitution will be given."
- Remember that starting with the substitution, say $u = x^2 + 1$, it helps to make x the subject, except in some cases where there's a trigonometric substitution, e.g. if $u = \sin x + 1$, but $\sin x$ appears in the expression to integrate, then we might make $\sin x$ the subject instead.
Differentiate and make dx the subject also, then ensure original expression is only in terms of new variable.
- Don't feel as if you need to memorise a separate formula for <u>parametric</u> volumes of revolution, since $dx = \frac{dx}{dt}dt$ clearly by the fact that the dt's cancel.
- You have to <u>**change the limits**</u> whenever you do either of: (a) parametric integration or (b) integration by substitution, because you're integrating in terms of a new variable.
- This is more use for STEP, but remember that $\int_b^a f(x)dx = -\int_a^b f(x)$, useful when the limits are the wrong way round.
- You can tidy things up sometimes using $-\int -f(x)dx = +\int f(x)dx$, since the -1 can be factored out the integral.
- For integration by parts, if you ever have to IBP twice, write the second integral as a separate result first before substituting it in after. This is to avoid sign errors and keep things tidy. e.g. Workings might be:
$$\int x^2 \cos x\,dx$$
$$u = x^2 \quad \frac{dv}{dx} = \cos x$$
$$\frac{du}{dx} = 2x \quad v = \sin x$$
$$\int x^2 \cos x\,dx = x^2 \sin x - \int 2x \sin x\,dx$$
"For $\int 2x\sin x\,dx$:"

- Sign errors when integrating/differentiating trig functions. Other than sin and cos, be careful about cot/cosec:
$$\frac{d}{dx}(\cot x) = -cosec^2 x$$
thus $\int cosec^2 x\,dx = -\cot x + C$
- A common one: Forgetting about the chain rule when integrating expressions of the form $(a+bx)^c$, see $\int(4-3x)^5 dx$ example.
- Remember that constants differentiate to nothing, i.e. $\frac{d}{dx}(\pi^2) = 0$ not 2π!
- Similarly $\ln c$ is a constant. $x\ln 2$ would differentiate to $\ln 2$.
- If $u^2 = x + 1$ is the substitution, you're doing unnecessary work if you then square root. Differentiating implicitly:
$$2u\frac{du}{dx} = 1$$
$$dx = 2u\,du$$
This is much much tidier!
- Forgetting to change your limits for either parametric integration or integration by substitution!
But note that in integration by substitution, once you've changed back to the original variable (probably x), you should use the original limits.
- Don't try and use integration by parts if you can use 'integration by inspection'.
e.g. For $\int x\,e^{x^2}$, then integration by parts would lead to a dead end.

		$u = 2x \quad \frac{dv}{dx} = \sin x$ $\frac{du}{dx} = 2 \quad v = -\cos x$ $\int 2x \sin x \ dx = -2x\cos x - \int -2\cos x \ dx$ $= -2x\cos x + \int 2\cos x \ dx$ $= -2x\cos x + 2\sin x$ $\therefore \int x^2 \cos x \, dx = x^2 \sin x - (-2x\cos x + 2\sin x)$ $= x^2\sin x + 2x\cos x - 2\sin x + C$ Note the nice double negative tidying up trick towards the end. • If you're solving $\frac{dy}{dx} = xy + y$, then you need the y (or whatever variable appears at the top of $\frac{dy}{dx}$) on the LHS. This is always achieved by a division or multiplication, which may require factorisation first: $\frac{dy}{dx} = y(x+1) \qquad \frac{1}{y}\frac{dy}{dx} = x + 1$ $\int \frac{1}{y} \ dy = \int x + 1 \ dx \qquad \ln\lvert y\rvert = \frac{1}{2}x^2 + x + C$ $y = e^{\frac{1}{2}x^2 + x + C} = Ae^{\frac{1}{2}x^2 + x}$ • Note in the above example, we let some new constant $A = e^C$ to help tidy things up. If we had $\ln x + C$ on the right-hand-side, we'd make $C = \ln A$ so that $\ln x + \ln A = \ln(Ax)$. Similarly if we had $\ln y = x + C$, and hence $y = e^{x+C} = e^x e^C$, we could make $A = e^C$. • In differential equations, ensure you separate the RHS into the form $f(x)g(y)$ first so that you are able to divide by $g(y)$, e.g. $\frac{dy}{dx} = x + xy \ \rightarrow \ \frac{dy}{dx} = x(1+y) \rightarrow \ \frac{1}{1+y} dy = x \ dx$ • In differential equations, if you're given initial conditions (note, often $t = 0$ is often implied for the initial condition), then it's generally easier to plug them in to work out your constant of integration sooner rather than later.	• For differential equations, ensure the variable at the top of the $\frac{d\square}{d\square}$ matches what you've moved to the LHS. e.g. If $\frac{dt}{dr} = r^2 t$ then it's the t you want on the LHS.

C4 Integration Cheat Sheet

$f(x)$	How to deal with it	$\int f(x)dx$ (+constant)	FormBk?
$\sin x$	Standard result	$-\cos x$	No
$\cos x$	Standard result	$\sin x$	No
$\tan x$	In formula booklet, but use $\int \frac{\sin x}{\cos x} dx$ which is of the form $\int \frac{kf'(x)}{f(x)} dx$	$\ln\lvert\sec x\rvert$	Yes
$\sin^2 x$	For both $\sin^2 x$ and $\cos^2 x$ use identities for $\cos 2x$ $\cos 2x = 1 - 2\sin^2 x$ $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$	$\frac{1}{2}x - \frac{1}{4}\sin 2x$	No
$\cos^2 x$	$\cos 2x = 2\cos^2 x - 1$ $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$	$\frac{1}{2}x + \frac{1}{4}\sin 2x$	No
$\tan^2 x$	$1 + \tan^2 x \equiv \sec^2 x$ $\tan^2 x \equiv \sec^2 x - 1$	$\tan x - x$	No
$cosec\ x$	Would use substitution $u = cosec\ x + \cot x$, but too hard for exam.	$-\ln\lvert cosec\ x + \cot x\rvert$	Yes
$\sec x$	Would use substitution $u = \sec x + \tan x$, but too hard for exam.	$\ln\lvert\sec x + \tan x\rvert$	Yes
$\cot x$	$\int \frac{\cos x}{\sin x} dx$ which is of the form $\int \frac{f'(x)}{f(x)} dx$	$\ln\lvert sin\ x\rvert$	Yes
$cosec^2 x$	By observation.	$-\cot x$	No!
$\sec^2 x$	By observation.	$\tan x$	Yes (but memorise)
$\cot^2 x$	$1 + \cot^2 x \equiv cosec^2 x$	$-\cot x - x$	No
e^x	Standard result	e^x	No
a^x	$y = a^x \quad \rightarrow \quad \ln y = x \ln a$ Then differentiate implicitly.	$\frac{1}{\ln(a)} a^x$	No
$\frac{1}{x}$	Standard result	$\ln x$	No
$\ln x$	Use IBP, where $u = \ln x, \frac{dv}{dx} = 1$	$x \ln x - x$	No

$f(x)$	How to deal with it	$\int f(x)dx$ (+constant)	FB?
$\sin 2x \cos 2x$	For any product of sin and cos with same coefficient of x, use double angle. $\sin 2x \cos 2x \equiv \frac{1}{2}\sin 4x$	$-\frac{1}{8}\cos 4x$	No
$\cos x\ e^{\sin x}$	Of form $g'(x)f'\big(g(x)\big)$	$e^{\sin x}$	
$\frac{x}{x+1}$	Use algebraic division. $\frac{x}{x+1} \equiv 1 - \frac{1}{x+1}$	$x - \ln\lvert x+1\rvert$	
$\frac{1}{x(x+1)}$	Use partial fractions.	$\ln\lvert x\rvert - \ln\lvert x+1\rvert$	
$\frac{4x}{x^2+1}$	Reverse chain rule. Of form $\int \frac{kf'(x)}{f(x)}$	$2\ln\lvert x^2+1\rvert$	
$\frac{x}{(x^2+1)^2}$	Power around denominator so NOT of form $\int \frac{kf'(x)}{f(x)}$. Rewrite as product. $x(x^2+1)^{-2}$ Reverse chain rule (i.e. "Consider $y = (x^2+1)^{-1}$" and differentiate.	$-\frac{1}{2}(x^2+1)^{-1}$	
$\frac{8x^2}{4x^2-1}$	Fraction top heavy so do algebraic division first. Then split into algebraic fractions as $4x^2 - 1 = (2x+1)(2x-1)$	$2x + \frac{1}{2}\ln\lvert 1-2x\rvert$ $-\frac{1}{2}\ln\lvert 2x+1\rvert$	
e^{2x+1} $\frac{1}{1-3x}$	For any function where 'inner function' is linear expression, divide by coefficient of x	$\frac{1}{2}e^{2x+1}$ $-\frac{1}{3}\ln\lvert 1-3x\rvert$	
$x\sqrt{2x+1}$	Use sensible substitution. $u = 2x+1$ or even better, $u^2 = 2x+1$.	$\frac{1}{15}(2x+1)^{\frac{3}{2}}(3x-1)$	
$\sin^5 x\ \cos x$	Reverse chain rule.	$\frac{1}{6}\sin^6 x$	
$\sin 3x \cos 2x$	Use identities in C3 formula booklet, $\sin 3x \cos 2x = \frac{1}{2}(\sin 5x + \cos x)$ Note: has never come up in an exam.	$-\frac{1}{10}\cos 5x$ $+\frac{1}{6}\sin 3x$	Sort of