Compound and Double Angle Trig Formulae Past Paper Questions

Key Cribs: *The aim is not to use these!*

Add any of your own "crib notes" to this sheet.

	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

$$\sin(3\theta) = \sin(\theta + 2\theta)$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$$

$$\sin\theta = tan\theta cos\theta$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

$$sin^2\theta = 1 - cos^2\theta$$

Make sure that you know, or can derive, the following:

Trigonometric Identities

Pythagoras's theorem

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta \tag{2}$$

$$\tan^2\theta + 1 = \sec^2\theta \tag{3}$$

Note that $(2) = (1)/\sin^2\theta$ and $(3) = (1)/\cos^2\theta$.

Compound-angle formulae

$$\cos(A+B) = \cos A\cos B - \sin A\sin B \tag{4}$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B \tag{5}$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B \tag{6}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B \tag{7}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \tag{8}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B} \tag{9}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \tag{10}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{11}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \tag{12}$$

Note that you can get (5) from (4) by replacing B with $-B$, and using the fact that $\cos(-B) = \cos B$ (cos is even) and $\sin(-B) = -\sin B$ (sin is odd). Similarly (7) comes from (6). (8) is obtained by dividing (6) by (4) and dividing top and bottom by $\cos A\cos B$, while (9) is obtained by dividing (7) by (5) and dividing top and bottom by $\cos A\cos B$. (10), (11), and (12) are special cases of (4), (6), and (8) obtained by putting $A = B = \theta$.

Sum and product formulae

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \tag{13}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \tag{14}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \tag{15}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \tag{16}$$

Note that (13) and (14) come from (4) and (5) (to get (13), use (4) to expand $\cos A = \cos(\frac{A+B}{2} + \frac{A-B}{2})$ and (5) to expand $\cos B = \cos(\frac{A+B}{2} - \frac{A-B}{2})$, and add the results). Similarly (15) and (16) come from (6) and (7).

Thus **you only need to remember (1), (4), and (6)**: the other identities can be derived from these.

June 2005

5. (*a*) Using the identity $\cos(A + B) \equiv \cos A \cos B - \sin A \sin B$, prove that

$$\cos 2A \equiv 1 - 2\sin^2 A.$$

(2)

(*b*) Show that

$$2\sin 2\theta - 3\cos 2\theta - 3\sin\theta + 3 \equiv \sin\theta(4\cos\theta + 6\sin\theta - 3).$$

(4)

(*c*) Express $4\cos\theta + 6\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

(4)

(*d*) Hence, for $0 \le \theta < \pi$, solve

$$2\sin 2\theta = 3(\cos 2\theta + \sin\theta - 1),$$

giving your answers in radians to 3 significant figures, where appropriate.

(5)

January 2006

7. (*a*) Show that

(i) $\dfrac{\cos 2x}{\cos x + \sin x} \equiv \cos x - \sin x, \quad x \ne (n - \frac{1}{4})\pi, \; n \in \mathbb{Z},$

(2)

(ii) $\frac{1}{2}(\cos 2x - \sin 2x) \equiv \cos^2 x - \cos x \sin x - \frac{1}{2}$.

(3)

(*b*) Hence, or otherwise, show that the equation

$$\cos\theta\left(\frac{\cos 2\theta}{\cos\theta + \sin\theta}\right) = \frac{1}{2}$$

can be written as

$$\sin 2\theta = \cos 2\theta.$$

(3)

(*c*) Solve, for $0 \le \theta < 2\pi$,

$$\sin 2\theta = \cos 2\theta,$$

giving your answers in terms of π.

(4)

January 2007

1. (*a*) By writing $\sin 3\theta$ as $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(5)

(*b*) Given that $\sin\theta = \frac{\sqrt{3}}{4}$, find the exact value of $\sin 3\theta$.

(2)

January 2008

6. (*a*) Use the double angle formulae and the identity

$$\cos(A + B) \equiv \cos A\cos B - \sin A\sin B$$

to obtain an expression for $\cos 3x$ in terms of powers of $\cos x$ only.

(4)

(*b*) (i) Prove that

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} \equiv 2\sec x, \qquad x \neq (2n+1)\frac{\pi}{2}.$$

(4)

(ii) Hence find, for $0 < x < 2\pi$, all the solutions of

$$\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = 4.$$

(3)

January 2009

6. (*a*) (i) By writing $3\theta = (2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$

(4)

(ii) Hence, or otherwise, for $0 < \theta < \frac{\pi}{3}$, solve

$$8\sin^3\theta - 6\sin\theta + 1 = 0.$$

Give your answers in terms of π.

(5)

January 2010

8. Solve

$$\text{cosec}^2\, 2x - \cot 2x = 1$$

for $0 \leq x \leq 180°$.

(7)

June 2010

1. (*a*) Show that

$$\frac{\sin 2\theta}{1 + \cos 2\theta} = \tan \theta.$$

(2)

(*b*) Hence find, for $-180° \leq \theta < 180°$, all the solutions of

$$\frac{2\sin 2\theta}{1 + \cos 2\theta} = 1.$$

Give your answers to 1 decimal place.

(3)

January 2011

3. Find all the solutions of

$$2\cos 2\theta = 1 - 2\sin\theta$$

in the interval $0 \leq \theta < 360°$.

(6)

June 2011

6. (*a*) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \qquad \theta \neq 90n°, \quad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

(3)

(ii) solve, for $0 < x < 360°$,

$$\text{cosec}\, 4x - \cot 4x = 1.$$

(5)

January 2012

5. Solve, for $0 \leq \theta \leq 180°$,

$$2 \cot^2 3\theta = 7 \operatorname{cosec} 3\theta - 5.$$

Give your answers in degrees to 1 decimal place.

(10)

June 2012

8.

$$f(x) = 7 \cos 2x - 24 \sin 2x.$$

Given that $f(x) = R \cos (2x + \alpha)$, where $R > 0$ and $0 < \alpha < 90°$,

(*a*) find the value of R and the value of α.

(3)

(*b*) Hence solve the equation

$$7 \cos 2x - 24 \sin 2x = 12.5$$

for $0 \leq x < 180°$, giving your answers to 1 decimal place.

(5)

(*c*) Express $14 \cos^2 x - 48 \sin x \cos x$ in the form $a \cos 2x + b \sin 2x + c$, where a, b, and c are constants to be found.

(2)

(*d*) Hence, using your answers to parts (*a*) and (*c*), deduce the maximum value of

$$14 \cos^2 x - 48 \sin x \cos x.$$

(2)

January 2013

6. (i) Without using a calculator, find the exact value of

$$(\sin 22.5° + \cos 22.5°)^2.$$

You must show each stage of your working.

(5)

(ii) (*a*) Show that $\cos 2\theta + \sin \theta = 1$ may be written in the form

$$k \sin^2 \theta - \sin \theta = 0, \text{ stating the value of } k.$$

(2)

(*b*) Hence solve, for $0 \leq \theta < 360°$, the equation

$$\cos 2\theta + \sin \theta = 1.$$

(4)

June 2013

6. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2 \theta + 3 \sec \theta = 2 \tan^2 \theta$$

You must show all your working. Give your answers in terms of π.

(6)

June 2014

3. (i) (*a*) Show that $2 \tan x - \cot x = 5 \operatorname{cosec} x$ may be written in the form

$$a \cos^2 x + b \cos x + c = 0$$

stating the values of the constants a, b and c.

(4)

(*b*) Hence solve, for $0 \leq x < 2\pi$, the equation

$$2 \tan x - \cot x = 5 \operatorname{cosec} x$$

giving your answers to 3 significant figures.

(4)

(ii) Show that

$$\tan \theta + \cot \theta \equiv \lambda \operatorname{cosec} 2\theta, \qquad \theta = \frac{n\pi}{2}, \qquad n \in \mathbb{Z}$$

stating the value of the constant λ.

(4)

June 2015

1. Given that

$$\tan \theta° = p, \text{ where } p \text{ is a constant, } p \neq \pm 1,$$

use standard trigonometric identities, to find in terms of p,

(*a*) $\tan 2\theta°$,

(2)

(*b*) $\cos \theta°$,

(2)

(*c*) $\cot (\theta - 45)°$.

(2)

Write each answer in its simplest form.

June 2016

8. (*a*) Prove that

$$2 \cot 2x + \tan x \equiv \cot x, \qquad x \neq \frac{n\pi}{2}, \qquad n \in \mathbb{Z}.$$

(4)

(*b*) Hence, or otherwise, solve, for $-\pi \leq x < \pi$,

$$6 \cot 2x + 3 \tan x = \operatorname{cosec}^2 x - 2.$$

Give your answers to 3 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

(Total 8 marks)