UKMT MENTORING SCHEME (Intermediate Level)
October 2010 (Sheet 1)
Questions

Welcome to the 2010 Intermediate Mentoring Scheme. Each month you will be set a sheet of eight questions. Mostly the problems will respond to ingenuity rather than special methods, though gradually over the course of the year some methods will be introduced. A really good problem should make you think "How on Earth do I do that?" And then after you have thought about it for a while and tried a few things, then a method of solution might gradually occur to you, so don't give up too easily! On the whole the earlier questions tend to be easier and the last couple will often be quite difficult. Ideally you should send your mentor or teacher ***full written solutions*** *by the deadline date.*

1. Suppose a and b are positive integers greater than 1 and that $\sqrt{a\sqrt{a\sqrt{a}}} = b$. Find the least possible value of $a + b$.

2. At the last but one test of the year, Ellie scored 98 and her average score increased by 1. At the last test she scored 70, so that her average score decreased by 2. How many tests did she do during the year.

3. Find all solutions to the system of equations
$$x^2 + y^2 = 25$$
$$x + y + xy = 19.$$

4. A coin is put on one square of a 5 × 5 grid. All the other squares are covered without overlap by 3 × 1 dominoes. Find with proof all the possible positions of the coin.

5. The points A, B, C, D are equally spaced (in that order) along a line, where $AB = BC = CD = 30$. The circles whose diameters are AB, BC and CD are drawn. The tangent from A to the circle diameter CD meets the middle circle (diameter BC) at E and F. Calculate the length EF.

6. There is more than one integer greater than 1 which, when divided by any integer k such that $2 \le k \le 11$, has a remainder of 1. What is the differences between the two smallest such integers?

7. In a tennis tournament, there are twice as many men as women. Each player plays exactly one match with every other player. If there are no ties and the ratio of the number of matches won by women to the number won by men is 7/5, find out the number of women playing in the tournament.

8. For what integer values of N is $\dfrac{N^2 - 71}{7N + 55}$ a positive integer?

Deadline for receipt of solutions: 29th October 2010